Solution 3 :

$1°/\ \lim\limits_{x\to+\infty}\dfrac{2x^2+x-3}{x^2+ax}=\lim\limits_{x\to+\infty}\dfrac{2x^2}{x^2}=2$.

$2°/\ \lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x-5}}{x}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2(1+\frac{2}{x}-\frac{5}{x^2})}}{2x}=\lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{1+\frac{2}{x}-\frac{5}{x^2}}}{2x}$

$=\lim\limits_{x\to-\infty}\dfrac{-x\sqrt{1+\frac{2}{x}-\frac{5}{x^2}}}{2x}=\lim\limits_{x\to-\infty}-\dfrac{1}{2}\sqrt{1+\frac{2}{x}-\frac{5}{x^2}}=-\dfrac{1}{2}$.

$3°/\ \lim\limits_{x\to-\infty}\sqrt{3x^2+x-1}+x=\lim\limits_{x\to-\infty}|x|\sqrt{3+\frac{1}{x}-\frac{1}{x^2}}+x$

$=\lim\limits_{x\to-\infty}-x\sqrt{3+\frac{1}{x}-\frac{1}{x^2}}+x=\lim\limits_{x\to-\infty}x(1-\sqrt{3+\frac{1}{x}-\frac{1}{x^2}})=+\infty$ car

$\lim\limits_{x\to-\infty}1-\sqrt{3+\frac{1}{x}-\frac{1}{x^2}}=1-\sqrt{3}<0$

$4°/\ \lim\limits_{x\to+\infty}\sqrt{x^2+x+1}-mx=\lim\limits_{x\to+\infty}x(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-m)$

or $\lim\limits_{x\to+\infty}\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-m=1-m$

- Si $m<1$: $\lim\limits_{x\to+\infty}\sqrt{x^2+x+1}-mx=+\infty$
- Si $m>1$: $\lim\limits_{x\to+\infty}\sqrt{x^2+x+1}-mx=-\infty$
- Si $m=1$:

$\lim\limits_{x\to+\infty}\sqrt{x^2+x+1}-x=\lim\limits_{x\to+\infty}\dfrac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x}=\lim\limits_{x\to+\infty}\dfrac{x(1+\frac{1}{x})}{x(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}=\dfrac{1}{2}$